Parametric design thinking: A case-study of practice-embedded architectural research

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The paper highlights aspects of a particular parametric design thinking (PDT) distilled from practice. It describes the components of PDT – cognitive model, design method and information processing model – that are critical to an efficacious, collaborative search for solutions to architectural problems. The aspect related to the information processing model is afforded a detailed examination, synthesising the state-of-the-art in practice and research. Lastly, case studies spanning six years trace the transfer of methods and knowledge from collaborations and prototypes into projects of Zaha Hadid Architects Computation and Design group (ZHCODE). These exemplify the role of a shared language of geometry and several process related aspects of parametric design critical to its success.

Keywords: parametric design, design research, research methods, digital design, computer aided design

The distinction between practice and research in architecture is often blurred, and routinely, the act of building is considered research in itself (Till, 2007). Operating in such a context, this paper will highlight a particular form of computationally augmented *design thinking* and its contributions to architectural knowledge. Highlighting the aspects of such a of design thinking, as excavated from a practice-embedded architectural research, drawing attention to the distinctions and synergy between practice and research in architecture, and case-studies of contemporary research and practice spanning six years, form the main contributions of the article.

1 Practice-embedded architectural research

The purpose of an architectural research within contemporary practice, it will be argued, is to generalize a relevant design thinking or method. Such thinking, in turn should be able to synthesize architectural knowledge that can be disseminated to a wider audience other than those involved in the project. The terms of architectural research and parametric design thinking are first briefly expanded and subsequently the practice-embedded architectural research is posited as yielding their combination.

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Architectural research is often misrepresented as mentioned previously. It may be interesting to consider that the Royal Institute of British Architects (RIBA), in its advisory related to R&D tax relief (RIBA, 2012), indicates that architectural research, must contribute directly to the advancement of science or technology. It further notes that science and technology are firmly rooted in the understanding of the physical, material world and its application.

However in a broader sense, architectural research can be considered as:

- a systemic generation of communicable knowledge (Archer, 1995), that involves a deliberate, planned enquiry posed in relation to a task at hand. This includes an explicit intention to make it intelligible to an appropriate audience.
- research *in*, *through* and *for* architecture (Frayling, 1993), with scholarship in architectural and construction history, development of software, gathering of reference and inspirational materials being examples of each respectively.
- as an *archaeology* of the tacit research that happens in practice (Till, 2007) and generalising it into communicable Research.

Design thinking as a form of solution based thinking, was originally posited in contrast and comparison to the so-called scientific method of knowledge creation (Archer, 1981; Cross, 1982; Simon, 1996). Specifically, design thinking thrives in contexts — termed *wicked problems* (Churchman, 1967; Rittel & Webber, 1973) — where the problem is either ill-formed and/or highly non-linearly connected with the solution i.e. situations where the linear method of problem description and problem solution (Archer, 1979; Dorst & Cross, 2001) might struggle to find solutions. Exact definitions of design thinking have been the subject matter of many symposia including the seminal *Conference on Design Methods* in 1962, 1965 and 1967. In short however, design thinking might be considered as a form of solution oriented thinking that arises from an intersection of a *cognitive model* of the activity — the broad picture of what is thought of as being done, *information processing models* and *methods or procedures of design*.

By extension then, *parametric design thinking* (PDT), the theme of the current issue, may be considered as a computationally augmented form of design thinking. It operates specifically within and in relation to the medium of Computer Aided Design (CAD). It seeks a symbiotic and synergetic relationship with the sciences, particularly computer science.

Practice Embedded Architectural Research (PEAR) arises from the combination of the three specific notions of architectural research noted previously with the understanding of design thinking above. The objectives of such an embedded research may then be thought of as excavating from practice, a design thinking that is communicable. Specifically, it focusses on excavating the components of design thinking – the cognitive model, information processing model and design methods. Attendant aspects of this endeavour include research *in* historic precedents and methods, and research *thorough* the making of prototypes, material and software.

2 A parametric design thinking

The article will proceed by describing and exemplifying the advantages of a particular form of parametric design thinking, distilled from the work of the Computation and Design group of Zaha Hadid Architects (ZHCODE) carried out in the past six years. The excavated cognitive model and design method is situated and the information processing model is devoted a more thorough investigation.

2.1 Cognitive model: darwinism in design

Philosopher Daniel Dennett extends the Darwinian evolutionary model, especially the *mimetic* approach espoused by Richard Dawkins (in his book *The Selfish Gene*), to the specific cultural fields of design technology and design intelligence (Dennett, 2009, n.d.). In investigating this inverse process of *creating* cultural and social artefacts, he compares the slow, trial and error, bottom-up, purposeless process of Darwinian evolution to that of a directed, rapid, top-down, purposeful search process of intelligent design. Specifically, he compares the design processes of a termite mound versus that of Antonio Gaudi's Sagrada Familia (Dennett, 2016). In effect, he is extending the search mechanisms and tools of *Darwinian spaces* (Godfrey-Smith, 2009) – a space of all possible organisms, to that of *Design spaces*, – a space of all possible (human made) designs. Thus, *Darwinism in design*, called an *intuition pump* by Dennett, can be thought of as a cognitive model in relation to the current discussion on design thinking.

The work of ZHCODE operates under such a Darwinian cognitive model. In projects and research strands where multiple authors contribute, the collective acknowledgement of a cognitive model or collective understanding of the activity everyone is involved in helps immensely. Further, such a model leads naturally to tracing a form of *genetic* tree of ideas, shapes and processes, as the project evolves (Figure 1). The computational medium is inherently well suited for such an evolutionary process, in that almost all digital content that is produced is explicitly driven by handful parameters, repeatable instructions and operations (Figure 2). This is true even when the shapes might be produced 'manually' (Figure 3). Thus, one of the critical aspects of such a cognitive model is that iterative design process must include and balance both exploratory and exploitative phases – a trait commonly expected in successful algorithms that are based on biological evolution (Črepinšek, Liu, & Mernik, 2013). The reason is that, an insufficiently broad search increases the probability of missing better solutions, whilst an inadequately aggressive optimization of competing solutions can also result in sub-optimal solutions.

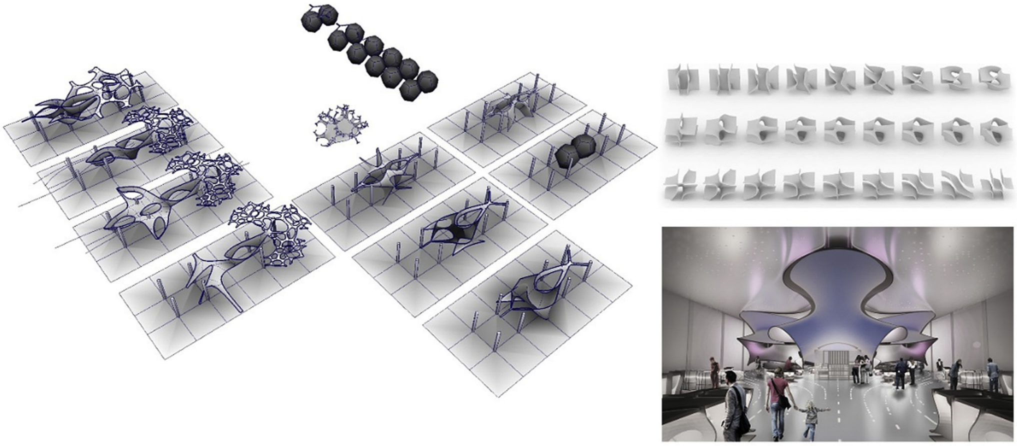

Figure 1 Darwinism in design ***Left*** Exploratory *search of topological variations* ***Right-Top*** Exploitative*, parametric search of topological variations. (See section 2.1 and 2.2 for more)* ***Right Bottom*** *submitted design.* ***Images:*** *courtesy of Zaha Hadid Architects*

Figure 2 Directed Search of solution space ***Top*** *exploratory search and exploited, refined option for* Volu *(see 4.2.1)* ***Bottom*** *exploratory search and exploited, refined option for* the Mathematics gallery *project (see 4.3.2).* ***Images:*** *courtesy of Zaha Hadid Architects*

Whilst the exact moment of switching from exploratory to exploitative mode is a matter of experience and available time, the cognitive model allows for the design team to acknowledge the necessity and anticipate the moment.

2.2 Design method: directed search of design-space

The natural design method that arises from the cognitive model above could be understood as a *directed* search of design space. A directed search seeks,

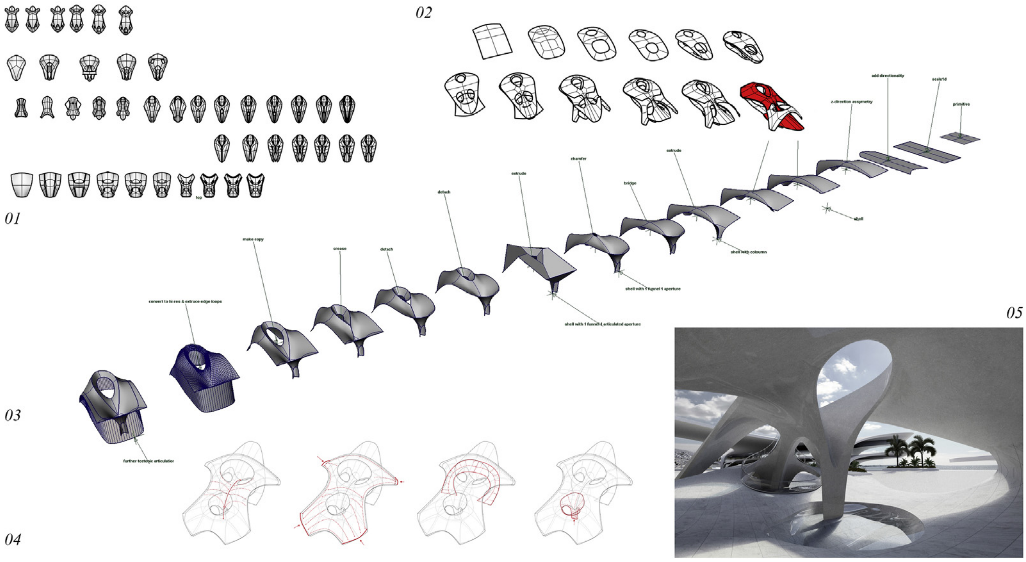

*Figure 3 Directed Search of solution space. **01** A* genetic *lineage of design options* ***02, 03*** *A sequence of geometric operations, amenable for hybridizing of options* ***04*** *Heuristics of structural behavioural* ***05*** *A* synthesized *solution.* ***Images:*** *courtesy of Zaha Hadid Architects and Block Research Group*

from among all possible solutions, a solution that is efficacious in its design and production and harmonious for human occupation. In other words, the aim of practice based research of ZHCODE is to build *well* i.e. to service the user in the Vitruvian sense (Wotton, 1624). It can be noted that, the efforts of the group in the past decade have focused on computational geometry as the mediating instrument to negotiate morphological, engineering and manufacturing logics. This is the focus of this article. The latter aspect of exploration of the societal purpose of architecture has, until recently, been left to accrued intuition of the designers: an intuition, to paraphrase Hillier and Hanson (1989), to "reproduce social circumstances in architectural form".

2.2.1 Exploration, exploitation and cumulativity

The advent of the computer and computer controlled machines in design and production of architecture, have both expanded the search space and expedited the search – i.e. aiding both exploration and exploitation. Computational technologies have already allowed for the assimilation of techniques and results from the natural and formal sciences into architectural design – mathematics of geometry, building physics, material chemistry, etc. The methods of enquiry from the two sciences, on the other hand, are not as widely assimilated or in the least, not as widely understood (Schumacher, 2016). In other words, the use of computational tools is increasingly widespread, but not the attendant *parametric design thinking* and espousal of the principles of scientific enquiry. Jon Elster, prominent social theorist, laments that such a preference for the consequents as opposed to the causal aspects is generally true in the social sciences (Elster, 2010). He argues that, the social sciences should aim to uphold the aspects of *cumulativity* and *irreversibility* that has served the natural and formal sciences so well — explaining ever more phenomena with time, building on prior work, generalizing results etc. As will be exemplified in the case studies (4), both the cognitive model and design method described here, support these aims — particularly that of *cumulativity*. Stated differently, a parametric design thinking that embraces *cumulativity* is better aligned with the general principles of scientific enquiry, and thus increasing the chances of a directed search being successful.

2.3 Information processing model: Computer Aided Geometric Design

Dennett (2009) argues that our ability to represent information, in words and symbols, is crucial to our ability to strategize towards seeking higher peaks (better solutions) within a design space. Thus, from an architectural design perspective, having a common language of geometric description can aid in collective problem solving by the principal stake-holders (client, architect, engineers etc.), building cumulative results and strategic planning.

Digital environments — termed *Design Explorers* (Kilian, 2006) — that enable rapid exploration of design space are key to both the efficient search of the design space and the development of novel outcomes. Further, design explorers that only allow structurally feasible and constructible geometries help vastly in narrowing the search space of architectural shapes. The constrained exploration helps focus efforts on developing other, and arguably, core aspects of spatial organization for human navigation and occupation. In other words, any degrees of freedom thus discovered, will be available for use to address problems related to social use of the building.

The use of geometric methods in the exploration of feasible forms has a rich history, particularly in the late 19th century (Evans, 2000; Witt, 2010). Complex 3D geometries were described using a widely disseminated protocol of 2D orthographic projections — a practice of *Descriptive Geometry* credited to French mathematicians, geometers and architects such as Gaspard Monge (Lawrence, 2011), Philbert De Lorme and Girard Desargues (Sakarovitch, 2003) among others.

Witt indicates the usefulness of Descriptive Geometry in the abstraction of mathematical knowledge into drawing instruments for specific types of complex geometry, manuals of construction for their physical realization in stone and timber, etc. Graphical means of structural analysis of the geometries so described, was also widespread — a famous example being their extensive use by Antonio Guadi (Huerta, 2006). Thus, a common language of geometry, lead to a profusion of innovation and assimilation of the material and construction technology within the building economy. In other words, geometric *Design explorers* of the period, not only embedded the structural stability of the design but also guaranteed its construction feasibility, thus contributing to its assimilation (Figure 4).

In the 20th century however, these geometric means of collaboration and integrated design began to strain under pressure from the advent of new materials and rational design (Straub, 1964), as also the ascendancy of new applied science and numerical methods of analysis (Picon, 1988). This has been noted to have contributed to a historic separation of architectural and engineering professions (Picon, 1988; Saint, 2007; Tessmann, 2008). All authors note that such a separation caused design exploration to become linear and fragmented as opposed to collaborative, circular and integrated.

The information processing model, described next in detail, aims to address this rupture.

3 Computer aided geometric design

Computational representations of objects in architectural design can be characterized into two paradigms – one *drawing* based and the other, *model* based. The drawing paradigm is popularly known as Computer Aided Design (CAD) and the model paradigm as Building Information Modelling (BIM). Each stems from seminal work of Ivan Sutherland (Sutherland, 1964, pp. 6–329) and Charles Eastman respectively (Eastman, 1975). Both *drawings* and *models* encode 2D and 3D geometry. A model however, contains meta-information about the encoded geometry – its material specification, role in processes of assembly, etc. On the other hand, the drawing paradigm, especially Computer Aided Geometric Design (CAGD), can support the creation of wider range of arbitrarily complex geometries, and its processing for Computer Aided Manufacturing (CAM). Further, an essential aspect of CAGD is the abstraction of complex physical phenomena and machine parameters associated with manufacturing method, into geometric properties and constraints. Famous examples include the automobile, aircraft, and shipbuilding industry motivating the development and use of Bezier curves and surfaces, physical splines, developable surfaces (Bezier, 1971; de Casteljau, 1986; Pottmann & Wallner, 1999; Pérez & Suárez, 2007; Sabin, 1971) (Figure 5) etc.

3.1 CAGD and CAE

Geometry creation environments and tools in CAGD, typically allow users to directly draw and manipulate smooth surfaces using a small set of so-called *control points* that are organised as a rectangular grid called a *control-net*. Intrinsic properties – position, curvature, tangent planes etc. – are defined everywhere on the surface and is a (smooth) function of the position of the

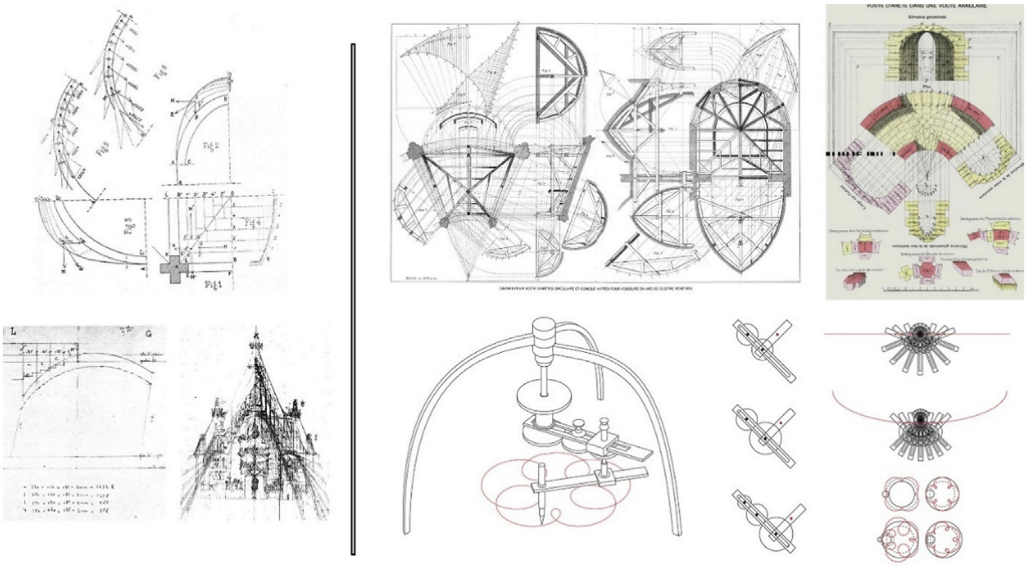

Figure 4 Statics and fabrication oriented Design Explorers. ***Left top*** *— Graphical analysis manual available in the late 19th century,* ***Left Bottom*** *— use of those methods by Antonio Gaudi in the design of Church of* Colonia Guell *and* Sagrada familia. ***Right*** *— 19th century Manuals for construction detailing in timber and stone, and drawing instruments, to realize complex geometries. (****Images****: Left Top and Bottom — from (Huerta, 2006), Right — from (Witt, 2010))*

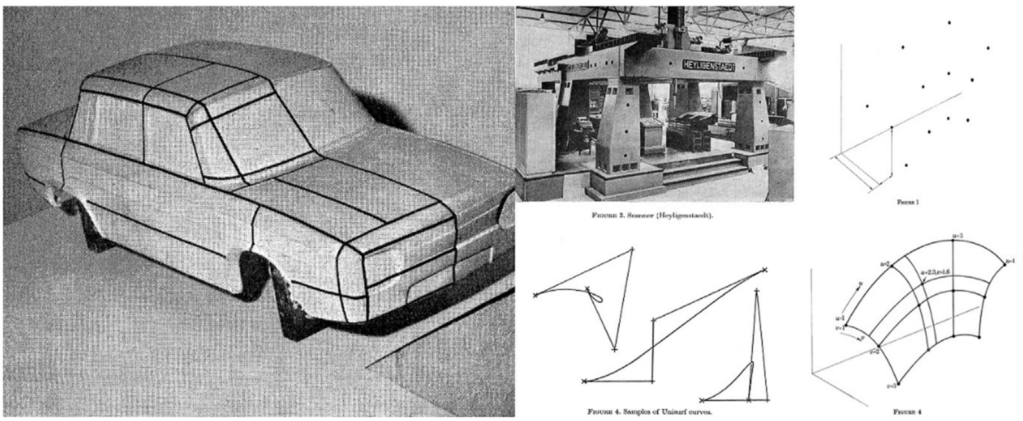

Figure 5 Computer Aided Geometric Design. ***Left and Middle columns****: Digital reconstruction process of master mould using Unisurf CAD system, in use at Renault Car Company around 1970: mark-up on clay master, 3D scanning, numerical input of points, creation of curve networks.* ***Right column*** *— similar system in use at British Aircraft Company (****Images*** left and middle from *— (Bezier, 1971). Right column — from (Sabin, 1971))*

control-points, and the parametric coordinates. Common examples are Bezier and Non-uniform rational Basis spline (NURBS) curves and surfaces. They allow for interactive and yet precise control over geometries. Such geometries are of high-fidelity and thus are well-suited for the use of CAM processes to manufacture them, and BIM to coordinate them with other building elements. Computer Aided Engineering (CAE) applications on the other hand, typically use a discretized representation of geometry. CAE surfaces are defined by a collection of triangles and properties of the surface are defined only on the vertices of this triangular net, called the Finite Element Mesh. Properties elsewhere are interpolated from their values at these vertices. Such discretization might be extended to solid elements such as tetrahedra. Further, the discretization has several criteria for being well-formed such as evenness of triangulation, constraints on the angles of the triangles etc.

This discrepancy between geometries that are suited for edit-friendly CAD applications and numerically biased CAE applications is one of the bottlenecks in a CAD-CAE design workflow. In the first instance, CAD geometries have to be converted to discrete representations and made suitable for numerical analysis. This is computationally and time-intensive. Further, once analysis is completed, mapping the results back unto the CAD design surface is also not well defined. There are no intuitive ways to manipulate the control-net of the CAD surface to affect the numerical values defined on the vertices of the CAE mesh. Thus, designers do not have any means to improve the structural soundness of the geometries — they are only able to visualise its structural performance (Whiting, 2012). Lastly, designers are not typically trained to understand the analytical and numerical processes of (structural) analysis or their results. In effect then, even though modern CAD applications contain modules that incorporate (structural) analysis within the design environment, the integration is at best time-intensive, non-iterative, opaque, and unidirectional. At worst, completely absent.

3.2 Integrating geometric design and structural design

Active strategies to overcome the lack of integration between geometric design and structural design are two-fold: defining and performing structural analysis directly on design geometries, and the inverse of designing with discrete, and analysis friendly geometries. The so-called Iso-geometric methods of structural analysis, falling in the former category, defines necessary analytical models directly using the control net of the NURBS surface i.e. structural properties are defined similarly to the intrinsic surface properties (Miki, Igarashi, & Block, 2015). The latter paradigm of surface design with discrete representation, though ubiquitous in the computer graphics and animation industry, is not as prevalent in architectural design. This mostly due to the lack of appropriate creation and manipulation tool-sets in popular CAD applications used by architects (Pottmann, Brell-Cokcan, & Wallner, 2006). Recent developments in the application of the mathematics of discrete differential geometry to architectural design — so called Architectural Geometry (Pottmann, 2007), has contributed to the popularisation of this paradigm. This paradigm is particularly popular in architectural projects with high geometric complexity

(Figure 6) (Veltkamp, 2010). These representations however, are currently not well supported in the BIM paradigm and thus present problems in construction coordination of architectural projects.

3.2.1 Form-finding

The so-called *Equilibrium modelling* methods attempt to find one or more, structurally appropriate geometries that satisfies equilibrium conditions, under the constraints of a prescribed stress-state of the surface, user-defined boundary conditions and external forces. Specifically, they aim to completely remove or minimise the bending stress of thin-shell surfaces, whilst being in static equilibrium. In other words, they synthesize surfaces that explicitly avoid bending and thus are well-aligned with fundamental tenet of lightweight structures (Bletzinger & Ramm, 2001; Schlaich & Schlaich, 2000). Procedures to find such surfaces are known as *form finding*.

Physical form finding proceeds by subjecting materials that cannot resist any bending forces such as chains, cloth and soap-films, to external loads and boundary conditions – usually hanging or wire boundaries. Computational methods replicate such explicit avoidance of bending, by making suitable assumptions in the structural equations. The Force Density Method (Schek, 1974) for example, produces surfaces that resist external loads by pure tensional internal stress, whilst the Thrust Network Analysis (TNA) (Block & Ochsendorf, 2007) produces a compression-only solution (Figure 7). A recent effort from Lachauer and Block (2014), extends the force density method to make it amenable to interactive modelling and constraint authoring of both pure tension and compression surfaces.

TNA and related efforts are a result of investigating the history of Graphic Statics (Culmann, 1875) and its extension to 3D modelling of structural equilibrium (Block, 2009) (Figure 8). They have contributed to several computational modelling methods to design free-form, self-supporting geometries (De Goes, Alliez, Owhadi, & Desbrun, 2013; Liu, Pan, Snyder, Wang, & Guo, 2013; Tang, Sun, Gomes, & Wallner, 2012; Vouga, Mathias, Wallner, & Pottmann, 2012). Additionally, they are amenable to the incorporation of manufacturing constraints (Panozzo, Block, & Sorkine-Hornung, 2013; Rippmann, Lachauer, & Block, 2012) and assembly aspects of digitally fabricated parts (Deuss et al., 2014; Schwartzburg & Pauly, 2013; Song, Fu, & Cohen-Or, 2012). These integrated digital explorers are already leading to visually telling, realised results (Rippmann et al., 2016; Rippmann & Block, 2013) (Figure 9).

3.2.2 Subdivision surfaces

One of the widely used geometric descriptions and technologies in the computer graphics and animation industry is the so-called subdivision surfaces (Catmull, 1974). This essentially involves the procedural generation of smooth

Figure 6 Realized projects with high geometric complexity in both their skin and structure. ***Left*** *– Opus, Dubai.* ***Right*** *– Heydar Aliyev Centre, Baku. (****Images****: courtesy of Zaha Hadid architects)*

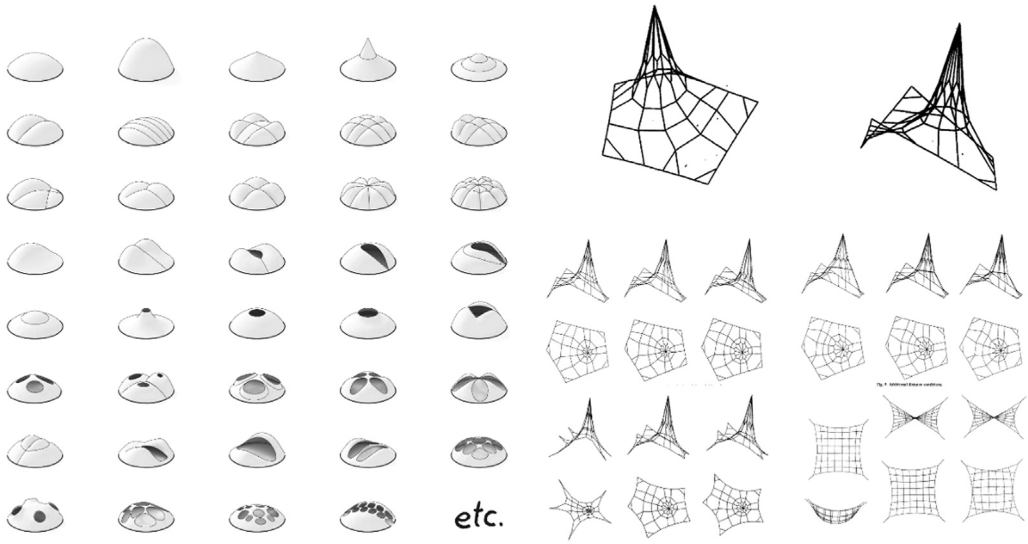

Figure 7 Equilibrium design space. ***Left*** *– various compression-only geometries generated using Thrust Network Analysis algorithm.* ***Right*** *– various tensile geometries generated using the Force Density Method algorithm. (****Images****: Left – from (Block, Lachauer, & Rippmann, 2014), Right – from (Schek, 1974))*

geometries via the subdivision of low-resolution input mesh geometry (Catmull, 1974). Most commercial Computer graphics and animation software have an extensive tool-set to aid the quick and interactive editing of the low-resolution input geometries and real-time visualization of the resulting subdivision surface. CAD packages such Rhinoceros™ and CATIA™ also supports these representations, though not extensively. Subdivision surfaces have the added benefit that they can be converted to Bezier patches (Stam, 2002; Stollnitz & Rice, 2005) and thus translated for BIM. In short, they are an ideal hybrid, suited for user-friendly manipulation, interfacing with structural form-

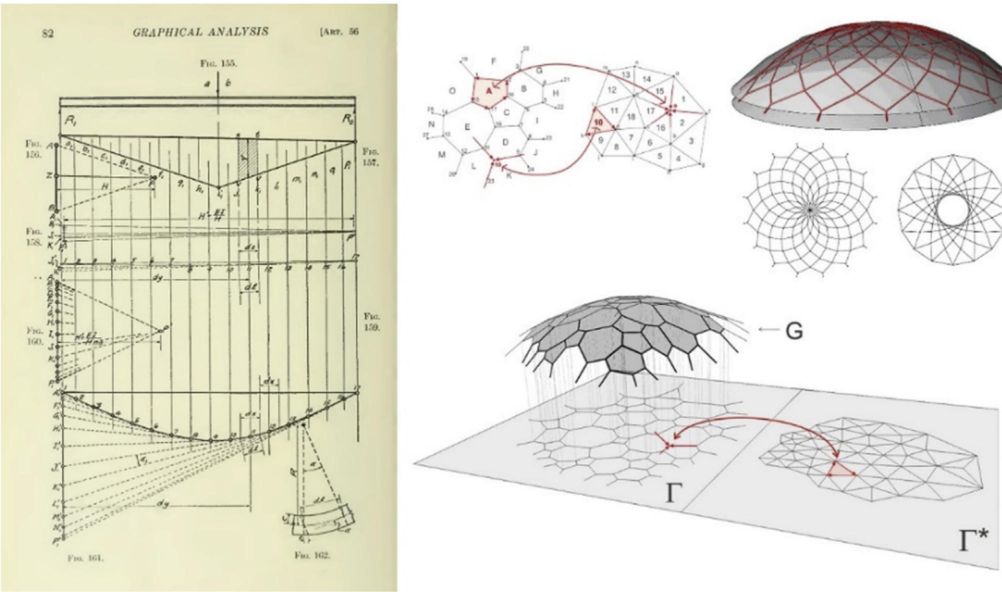

Figure 8 Graphical methods of finding equilibrium shapes. ***Left*** *− 2D geometrical construction of a catenary arch.* ***Right*** *− 3D construction of funicular geometry by Thrust Network Analysis. (**Images**: Left from (Wolfe, 1921), Right from (Block & Ochsendorf, 2007))*

finding and for translation in to BIM representation for down-stream coordination. The benefits of subdivision surface based modelling in architectural form generation have been previously established (Bhooshan & El Sayed, 2011; Shepherd & Richens, 2010). There have also been prior attempts to combine them with numerical modelling techniques to physically realise them with fabric (Bhooshan & El Sayed, 2012),Curved-Crease folded metal (Bhooshan, 2016b;

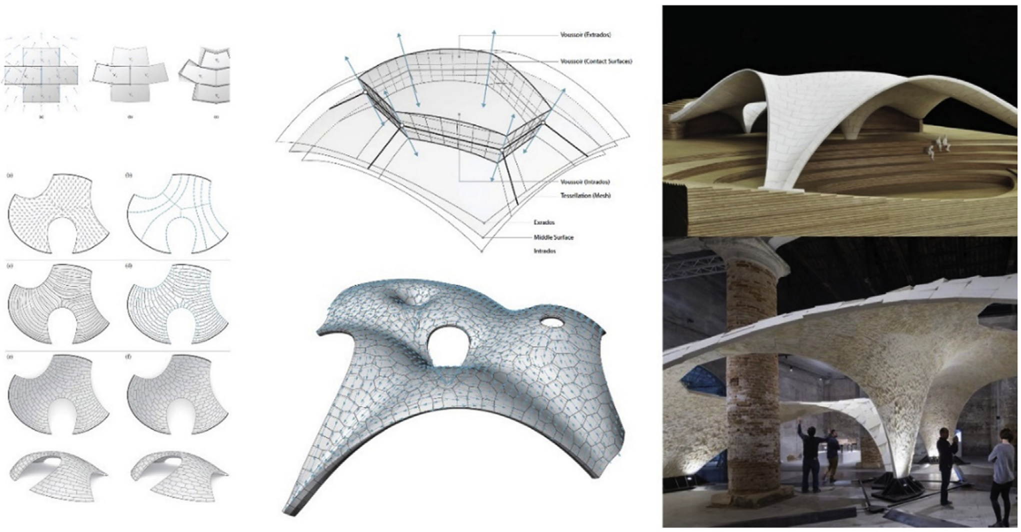

*Figure 9 Force-flow aligned tessellation of funicular geometries and built results. (**Images** from (Rippmann, 2016))*

Louth, Shah, Bhooshan, Reeves, & Bhooshan, 2015) and 3D printing (Bhooshan, 2016a). Some of these are described further in 5.

4 Case studies

The case studies, spanning six years, trace the accumulation and transfer of methods and knowledge from *collaborations* (4.1) and *prototypes* (4.2) unto *projects* (4.3) of ZHCODE. These case-studies, where possible, will explicitly illustrate the use of the Darwinian cognitive model (2.1) and the resulting design method (2.1). In particular however, they will exemplify the role of a shared language of geometry and several process related aspects of parametric design critical to knowledge accruement as discussed in 2.3 & 3.

4.1 Collaborations

4.1.1 Block research group and discrete funicular structures

The Block Research Group (BRG) at ETH Zurich takes a multi-disciplinary approach to the computational exploration of structural forms and appropriate methods for their construction. They specialize in graphical methods for the equilibrium design of complex discrete shells. The BRG has been a collaborator of ZHCODE, and have introduced force-driven computational tools and methods for exploration of freeform curved surface structures. ZHCODE have implemented some of the algorithms as revealed (4.2.3) or developed by BRG (3.2.1). Thus the collaboration has improved mutually, the general awareness of structurally plausible and sound geometries and methods of solving particular manufacturing constraints.

For example, a particular collaboration, explores iterative search methods for the form finding of discrete funicular structures under constraints resulting from requirements of curved-crease folded (CCF) moulds (Figure 10). It integrates the two-step form finding process described in Louth et al. (2015) into a unified procedure and thus overcomes the design difficulties described there. As noted in Louth et al. (2015), there is a general compatibility between compressive structures and CCF moulds. However, there is also an inherent negotiation between the local curvatures of the equilibrium shape and the cross-sectional depth of the mould so formed. The collaboration resulted in an easy-to-implement search optimization method, based on the variational extension of the Force Density Method described in Lachauer and Block (2014) that is compatible with interactive shape modelling and satisfies specific static equilibrium and mould-making constraints.

4.1.2 University of Bath and iterative methods in computational geometry

The Department of Architecture and Civil Engineering at the University of Bath have had significant impact on the methods of computational geometry

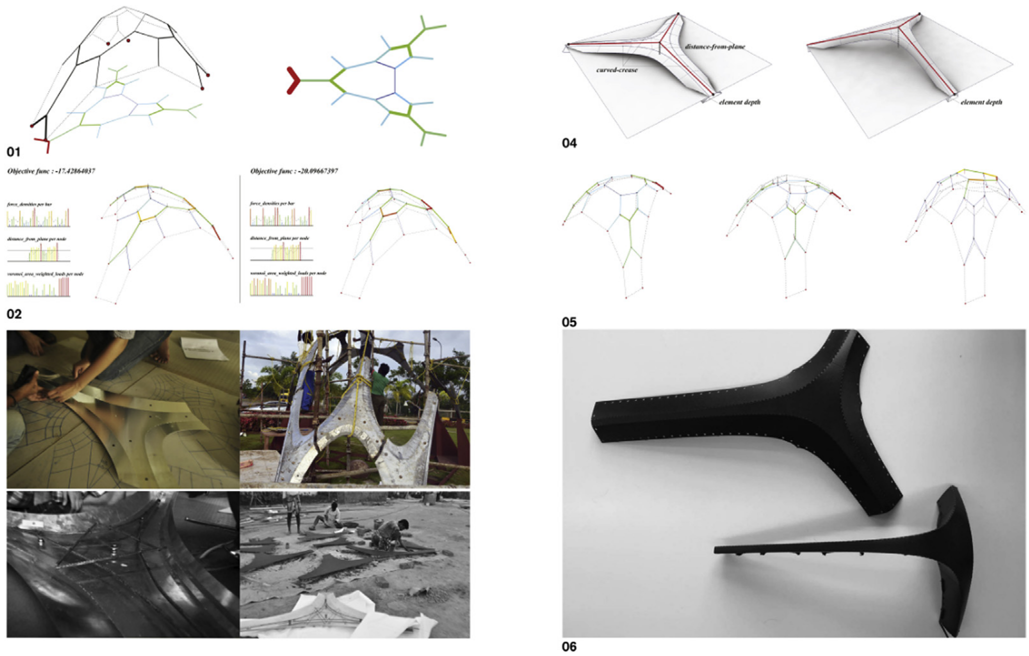

Figure 10 Augmenting the Force Density structural design algorithm to include manufacturing constraints. ***01*** *− Form-finding using the Force Density Method.* ***02*** *−Curved Crease Folding (CCF) of moulds, casting concrete and assembly.* ***04*** *− Expressing beam depth requirements of CCF moulds, geometrically.* ***05*** *− Various stages of the modified algorithm.* ***06*** *− Effect of 04, seen physically. (**Images**: 01−05 − from (S Bhooshan, Van Mele, & Block, 2015), 06 − courtesy Zaha Hadid Architects)*

design employed in several leading architectural practices including Foster and Partners (F&P), and Zaha Hadid Architects (ZHA). In particular, they have revealed implementation details of an originally not-so-simple method of computational simulation − the so-called Dynamic Relaxation method.

Dynamic relaxation (DR) as originally developed by Day (1965) and extended by Barnes (1999), is a computational method used to find equilibrium shapes of geometries subjected to (axial) forces i.e. form-finding. It has been extensively used find the shapes of cable-nets, and fabric membranes subjected to tensile forces − the so called minimal-mean-curvature-nets (M-surfaces) (Wakefield, 1999). Famously, Dr Williams from University of Bath used a modification of the method to design the roof of the British Museum, London for F&P (Shepherd & Williams, 2010). The method shares similarities with the particle-spring method of simulating various deformable surfaces such as cloth (Baraff, Witkin, & Kass, 1997; Bhooshan, Veenendaal, & Block, 2014), commonly used in computer graphics applications.

The method has also been employed in a geometric setting as opposed to its original setting of form-finding. For example Gauss (2014), modifies the method to get a planar mesh from an initially non-planar mesh. Similar curvature-based modifications can produce developable surfaces (D-surfaces) from initial non-developable surfaces. This aspect of using DR as a framework to variably produce M-surfaces or D-surfaces has particular architectural benefits: M-surfaces can be realised by stretching or tailoring sheet material such as fabric, where-as D-surfaces can be formed from sheet material such as metal (Figure 11).

ZHCODE have employed variants of this method to handle both the approximate form-finding (4.3.1) and to solve geometric and construction related constraints in several of their prototypes and projects. It can be noted that, the initial versions of the popular software add-on called *Kangaroo*, that operates within the CAD software of Rhinoceros™ also employed and extended this method for general use in problems of computational geometry.

4.1.3 Design Research Laboratory and exploratory design search

Lastly, it must be acknowledged that the several aspects of the research and projects presented here trace a lineage to the exploratory efforts of students, and researchers at the Design Research Laboratory (DRL), London. In particular exploration into the architectural use of subdivision surfaces, their realisation using textiles, physical studies in curved-crease folding etc. provide the background, inform and motivate the research and projects presented in this paper.

4.2 Prototypes

4.2.1 Volu, a pre-fabricated pavilion

The design brief of the project was to manufacture an economical and prefabricated pavilion composed exclusively with off-the-shelf parts and/or laser-cut components.

Modelling methods that take construction and fabrication into consideration are increasingly valued in delivering freeform geometries utilizing existing manufacturing pipelines (Jiang, Tang, Tomičí, Wallner, & Pottmann, 2015). This project documents the practical application of various computational methods towards effective, time-bound, collaborative and practical realization of complex geometries. The design pipeline used, builds upon the sub-division mesh modelling approach (Bhooshan & El Sayed, 2011; Shepherd & Richens, 2010). The development of the layout of the structural skeleton is informed by Topology Optimization (TO) (Bendsoe & Sigmund, 2013; Rozvany, 2001). The gradated material densities associated with a TO solution, are interpreted as discrete bar-node elements, that serve as a general arrangement suitable for further optimization under spatial and fabrication constraints (Beghini, Carrion, Beghini, Mazurek, & Baker, 2014). This re-interpretation is manually

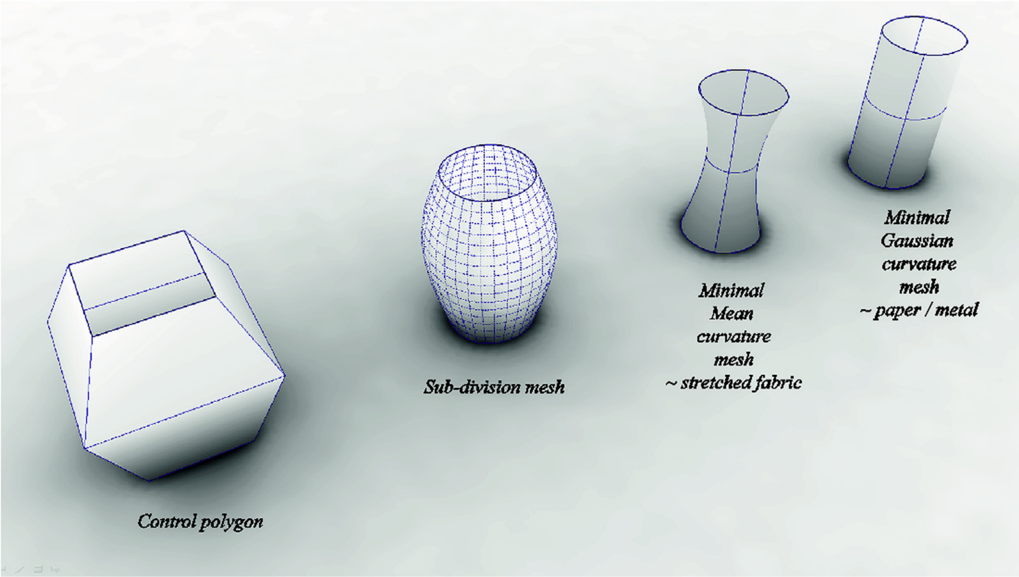

*Figure 11 A edit-friendly modelling paradigm, whereby a user-specified coarse mesh is algorithmically subdivided and perturbed to states of minimal Mean or Gaussian curvature. This makes the geometry suitable for realization with fabric-like or sheet metal-like materials respectively. (**Image** from (Bhooshan, 2016b))*

reconstructed from the TO result, and represented as a predominantly *quad (faced) mesh*.

The critical fabrication constraints, expressed geometrically, were to ensure that the joint-geometries were torsion-free or *extrude-able*, and the surfaces − top and bottom covers, and walls of the cells − were *Developable (*Figure 12). *Extrudability* of the vertices ensures that the edges of the mesh can be uniformly offset, and thus the derived beam network can be of uniform thickness. This makes the edge-layout amenable for realization using standard box-sections of aluminium. The chosen method of forming sheet material was to kerf-cut and bend steel for the top and bottom covers and plywood for the cell walls. These fabrication constraints along with other spatial and aesthetic requirements were handled by implementing a flexible constraint solver similar to Attar et al.(2010). Further, A so-called *Edge-Offset Mesh (EO Mesh)* (Liu & Wang, 2008; Pottmann & Wallner, 2008) is procedurally derived from the *quad-mesh*. Subsequently, we utilize a Planar-Quad mesh strip (PQ-Strip) representation of a developable surfaces (Kilian et al., 2008) and projection-based dynamics (Bouaziz, Deuss, Schwartzburg, Weise, & Pauly, 2012) to minimally perturb the vertices of *EO-mesh* towards locations that simultaneously satisfy the two critical fabrication criteria: *Extrudability* of the nodes and *developability* of the surfaces. The resultant *EO mesh* is utilized for downstream generation of structural and cladding components that are, at most, singly curved and thus allowing for the bending from flat sheet materials.

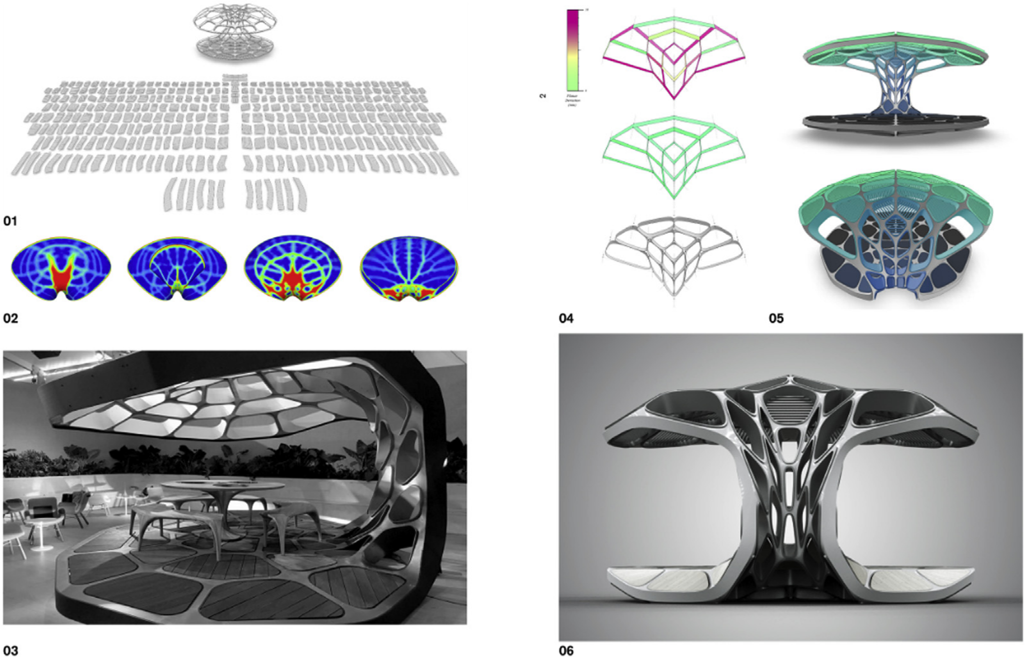

Figure 12 Volu, *a pre-fabricated pavilion.* ***01*** *Unrolled beam covers to be kerf-cut and folded.* ***02 Results*** *from Topology Optimization algorithm 03 Photograph.* ***04*** *Solving fabrication constraints geometrically.* ***04*** *Various stages of the algorithm 05 Gradient colour scheme.* ***06*** *Effect of 04, seen physically. (**Images**: all images courtesy Zaha Hadid Architects)*

4.2.2 Arum – self-supporting assembly of curve-crease folded panels

This sculpture is a result of research in the design and fabrication of a self-supporting, multi-panel installation for the Venice Biennale 2012. It operates against the backdrop of the exciting potentials that the field of curved-crease folding offers in the development of curved surfaces that can be manufactured from sheet material. The two main challenges were developing an intuitive design strategy and production of information adhering to manufacturing constraints. The essential contribution of the sculpture is a method for designing curve-crease geometries that could negotiate the multiple objectives of ease of use in exploratory design, and manufacturing constraints of their architectural-scale assemblies (Figure 13). There are several seminal design and art precedents within this field – Richard Sweeney (Sweeney, 2006), Huffman (1976) Erik Demaine (M. Demaine, n.d.) etc. Most of the precedents projects and available literature on design methods highlight the difficulty in developing an intuitive, exploratory digital-design method to generate feasible 3D geometries. The initial survey of methods included both the simple and common method – the method of reflection (Mitani & Igarashi, 2011) – and the involved Planar-Quad-meshes and optimization-based method (Kilian et al., 2008). Most methods, including the two above, presented difficulties towards incorporation within an intuitive, and parametric early-stage-digital-design method, with the first one proving difficult to explore variety of generalized solutions free of prior assumptions and the second one being elaborate involving scanning of physical paper models, proprietary optimization algorithms etc. For an extensive overview on the precedents, and computational methods related to curved crease folding, the reader is referred to a survey (Demaine, Demaine, & Koschitz, 2011) and a recent dissertation from University of Bath (Bhooshan, 2016b).

4.2.3 Freeform developable skeletons

Many of the challenges in modelling and fabricating spatial network structures stem from geometric complexities at structural nodes. These elements are often treated as unique components within a larger standardized assembly that are both time consuming to resolve and expensive to fabricate (Pottmann et al., 2015). This tends to motivate the use of repetitive elements which significantly limits design freedom.

A method for modelling a class of freeform spatial structures whose inherent geometric properties greatly simplify the design and fabrication of structural nodes was developed. The method is derived by combining a historic theorem in geometry – Varignon's theorem (Coxeter & Greitzer, 1967) and contemporary theorem in developable surfaces (Lang & Röschel, 1992) (Figure 14). Through this approach, complex joinery is replaced by segments of singly curved sheet/plate material that are formed through standard low-tech bending processes. This allows for rapid fabrication via relatively inexpensive 2-axis CNC cutting technology. The design space of viable spatial network structures is expanded considerably as component variation has minimal impact on cost within this fabrication/assembly pipeline.

Further, building upon the notion of reciprocity between form and force diagrams (Maxwell, 1870), the method represents a structural network via its dual – a volumetric mesh composed of irregular polyhedra with shared faces. While the structural significance of this approach has already been detailed by Akbarzadeh, Van Mele, & Block (2015) (4.1), this prototype shows that it also has advantages related to fabrication and assembly.

4.3 Projects

Two on-going projects of ZHCODE – one, a very large scale cluster of shell structures at an undisclosed location and the other, and the recently completed gallery for mathematics at the Science Museum in London – best exemplify the benefit of the research programme outlined previously. These include capacities to assimilate historic knowledge, enable and benefit from collaboration and iteration, amenability to digital fabrication etc.

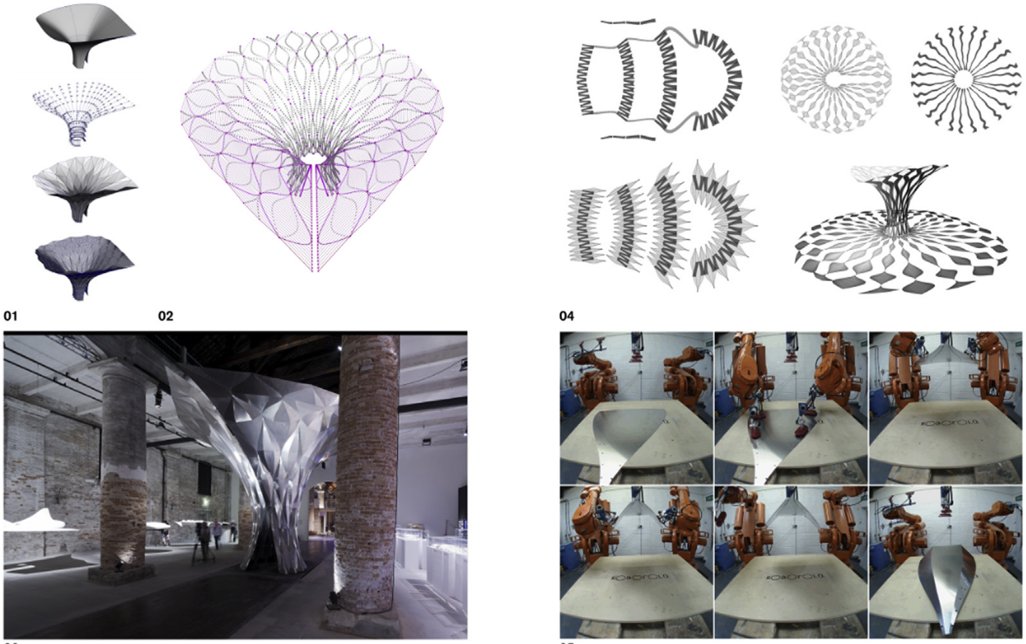

Figure 13 Arum*, a Curved Crease Folded Sculpture.* ***01, 02*** *layout and geometry of panels derived from base subdivision surface geometry.* ***03*** *Photograph.* ***04*** *Unrolled layout of all the panels.* ***05*** *Robotic folding of the panels. (****Images****: all images of courtesy Zaha Hadid Architects)*

4.3.1 *An institutional building, undisclosed location*

The design of shell-clusters of this project employed subdivision surfaces (3.2.2), along with the accrued knowledge of manipulating them algorithmically, the range of structurally beneficial curvatures to expect etc. (see 3.2.2, 4.1.1). The coarse mesh, description of the design geometry was particularly handy. During the competition stages, cloth-based simulations in Maya were used to visualise how far from a loaded surface, the subdivision surface would be. This practice followed into the early project design stages. The final shape was decided by the engineers – AKT II – using the parametric definition of the coarse mesh provided by the architects, combined with their structural analysis components. The subdivided mesh, was subjected to iterative optimisation for earthquake and vertical loading, whilst in the meantime the architects used the edit-friendliness of the coarse mesh to manipulate it to be aligned with the simulated stress-field/force flow, spatial concerns etc (Figure 15).

4.3.2 *Gallery for mathematics, Science Museum London*

The geometry and materialisation of the central organising features of the gallery are a result of both practical transfer of knowledge across disciplines and also a lineage fabric structures that ZHA had undertaken in the past. The geometry of these constructs – so called minimal surfaces – was intensively

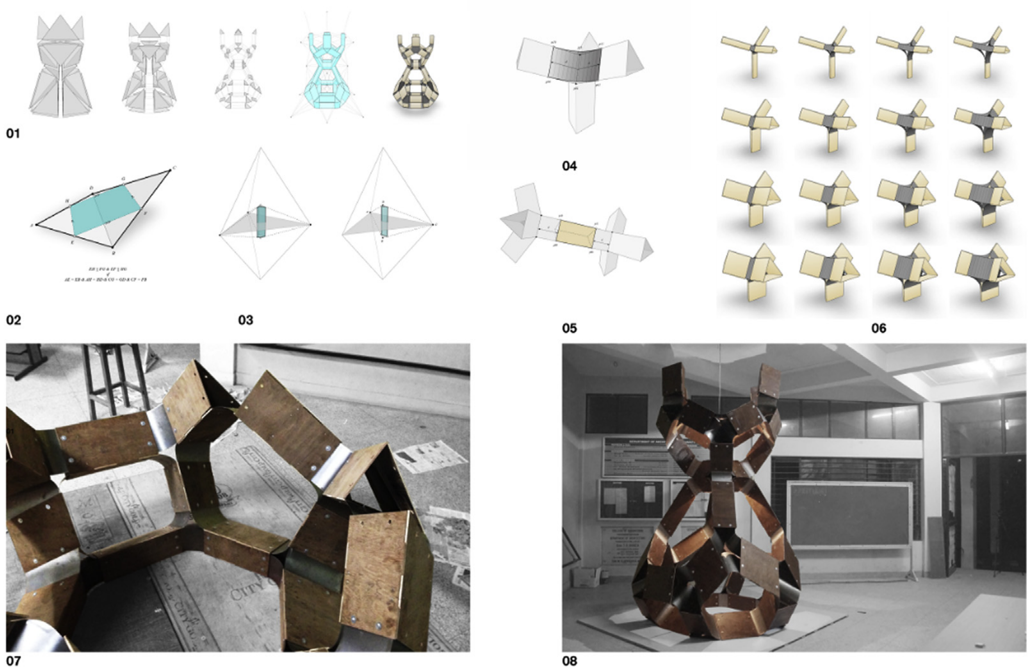

Figure 14 Freeform, developable skeletons ***01*** *Procedural generation of the skeleton from user-specified polyhedra.* ***02*** *Varignon's theorem.* ***03*** *Application of Varignon's theorem.* ***04, 05*** *Ensuring developability using theorem from (Lang & Röschel, 1992)* ***06*** *Nodal variation. (**Images**: courtesy Zaha Hadid Architects)*

studied by pioneering architect-engineer Frei Otto. He studied them physically as soap-films that form against a given wire boundary. These geometries have also been studied mathematically (Brakke, 1992). Their computational *generation* or form-finding process (3.2.1) usually employs one of two popular methods – the Force density method (Schek, 1974) and the Dynamic relaxation method (Day, 1965). These seminal methods have been made more accessible to architects and engineers alike by research institutions like Block Research Group (Adriaenssens, Block, Veenendaal, & Williams, 2014) (4.1.1) and University of Bath (Bak, Shepherd, & Richens, 2012; Williams, 1986) (4.1.2). Their architectural materialisation as stretched cable and fabric forms has been studied by several architectural and engineering firms. Prominent prior examples include the seminal Munich stadium by Frei Otto, and the temporary Serpentine Pavilion (London), the Magazine restaurant (London), the interactive *Parametric Space* installation (Copenhagen) by Zaha Hadid Architects etc. Thus the latest manifestation of such structures in the gallery is a result of a long history of prior experience and historically assimilated and transferred research (Figure 16).

Additionally the gallery has several moments of 'pause' including fourteen benches – designed as cast, ultra-high performance concrete benches. The

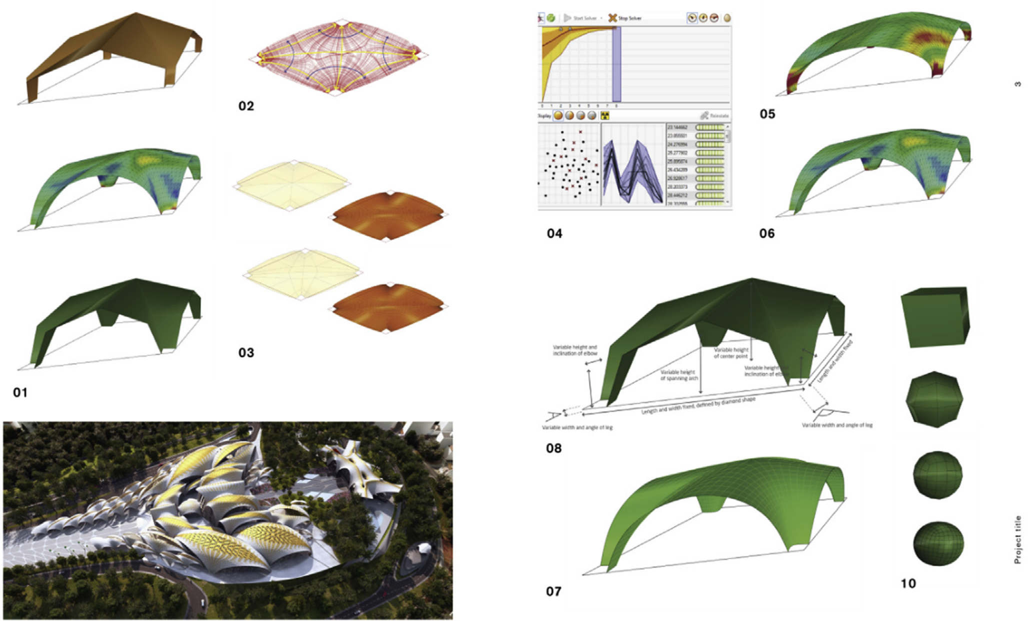

*Figure 15 Subdivision surfaces combined with Finite Element Analysis. **01, 07, 08** parametrically defined coarse mesh, and corresponding subdivision mesh and analysis results. **02** Stress field visualized on the surface. **03** Manipulating coarse mesh to align with stress field. **04, 05, 06** Finite Element Analysis and optimization. **10** Schema of subdivision surfaces (**Images**: courtesy AKT-II, London)*

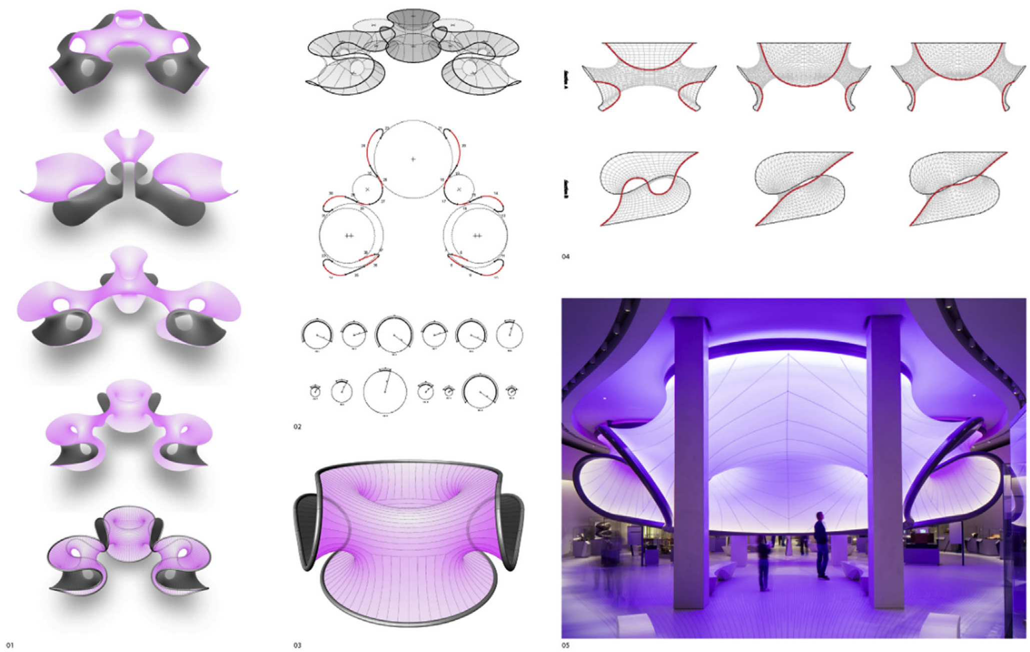

*Figure 16 Fabric structures of the mathematics gallery. **01** Multiple design iterations enabled by quick CAGD workflow **02** Bespoke CAGD tools ensured the edge-pipes could be bent physically. **03** Seam layout (in collaborations with Base Structures and Mark White). **04** Comparison of CAGD and Engineered Geometries. **05** Realised structure. (**Images**: courtesy Zaha Hadid Architects)*

shape and physical production of this furniture benefitted from methods of Descriptive geometry (2.3), research in the related but more complex geometries of Curved-crease folding (4.2.2), and collaboration with a state-of-the-art robotic company specialising in hot-wire-cutting of foam (McGee, Feringa, & Søndergaard, 2013) to produce the moulds for the cast concrete (Figure 17).

5 Conclusions

The article highlighted the need for architectural design and practice to follow a *research programme* (Lakatos, 1978), as opposed to ad-hoc solutions to design tasks. Imre Lakatos, a philosopher of mathematics and science, used the word – *research programme* – both in the pragmatic terms of cultivating experience and also the philosophical sense of maintaining a set of core-beliefs. Further, the three aspects of parametric *design thinking were* distilled from such an architectural research programme and exemplified with case studies. The case-studies in themselves showed that these features are enabling a dense network of cumulative, collaborative research involving academic institutions, professional firms and embedded research groups, to effectively realize architectural projects. In other words the cumulative research is enabling a successful outcome to the directed search of design-space.

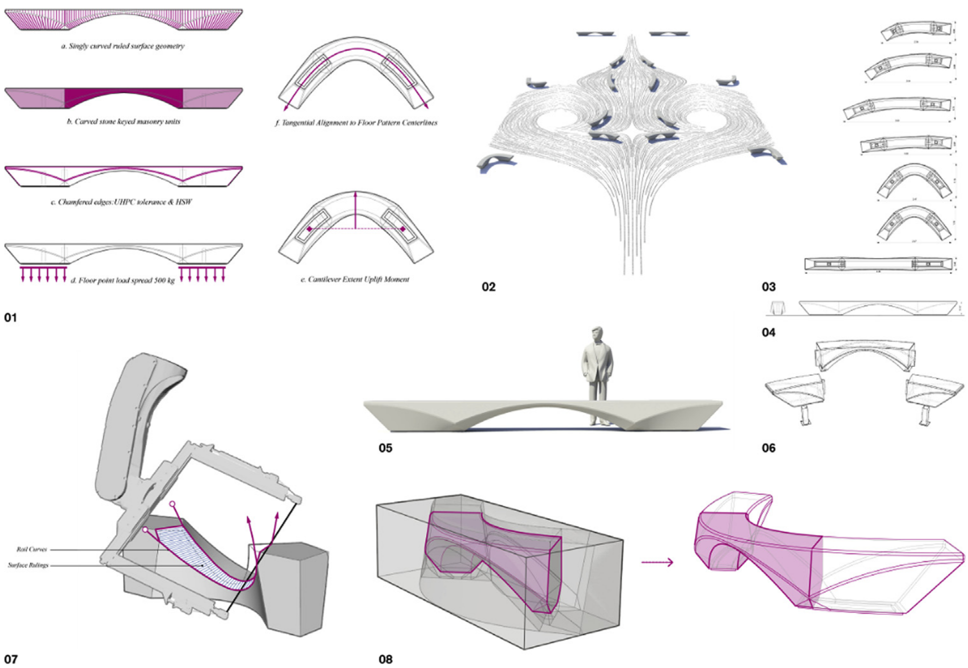

Figure 17 Designing with ruled surfaces and robotic hot-wire cutting of shapes ***01*** *Use of Ruled surfaces in CAGD to define the geometry.* ***03, 04, 06*** *Design development, drawings, schedule etc., using a bespoke BIM tool.* ***02, 05*** *CG images of all the benches and an individual.* ***07, 08*** *Robotic hot-wire cutting of ruled geometry. (**Images**: courtesy Zaha Hadid Architects)*

The benefits of following a research program are thus two-fold. It may be noted here that, as trivially obvious as both aspects might seem, it is far from de facto in current architectural practice. On the one hand, aligning practice-embedded research with established research trajectories allows for practitioners to focus their efforts on the social implications of the built environment, which has, despite its importance and impact, hitherto received rather scant attention from designers (Hillier & Hanson, 1989). Early evidence of this can be discerned in the design of the mathematics gallery (Figure 18). Additionally, this alignment is also mutually beneficial to the researchers in that their work can be motivated by and tested against its application in the field.

On the other hand, practice can benefit both from historically accrued knowledge and from the significant progress being made by researchers in the fields of architectural design (Thomsen, Tamke, Gengnagel, Faircloth, & Scheurer, 2015), computational geometry (Adriaenssens, Gramazio, Kohler, Menges, & Pauly, 2016), structural design ("IASS Symposium," 2015), robotic manufacture (Reinhardt, Saunders, & Burry, 2016) etc. This cultural accumulation and transmission is well-known to be critical to human evolutionary success and thus prudent to aim for.

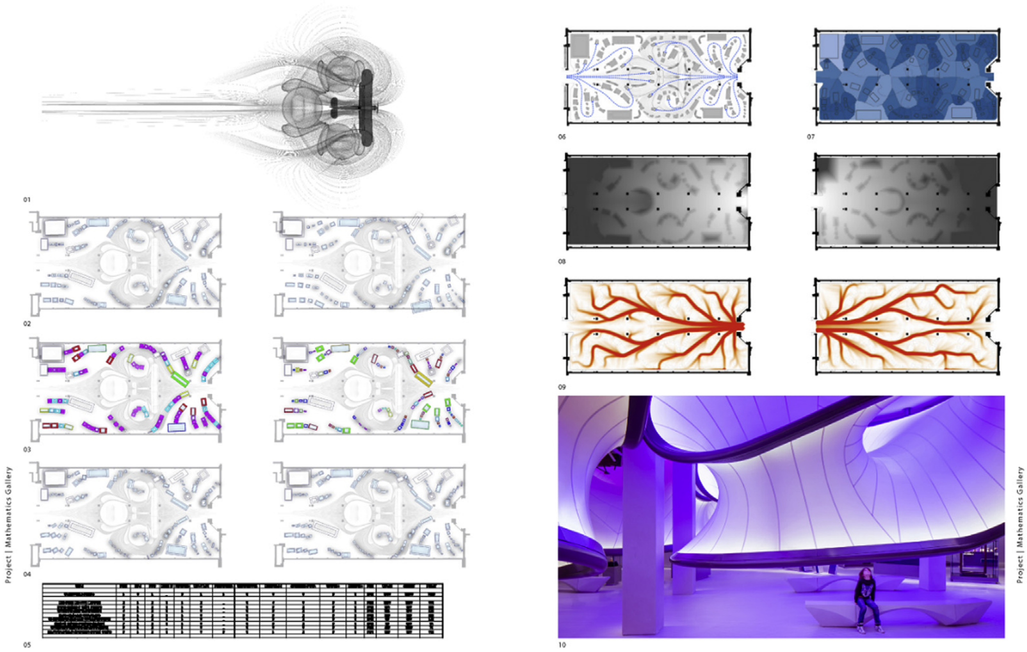

Figure 18 A computational approach to user navigation, occupation and ergonomics. ***01*** *Concept diagram of air-flow around the aeroplane.* ***01, 02, 03, 04*** *Data driven approach to accommodate layout changes per curatorial vision and other constraints.* ***05*** *Snippet of the detailed list of 130 objects.* ***05–09*** *Bespoke tools to analyse user, navigation and dwell experience* ***10*** *View of the gallery. (**Images**: courtesy Zaha Hadid Architects)*

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